

THE
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TABLE OF CONTENTS

ARTICLES

| | |
|---|---|
| The problem of logical constants. by MARIO GÓMEZ-TORRENTE | 1 |
|---|---|

COMMUNICATIONS

| | |
|--|----|
| Groups and algebras of binary relations. by STEVEN GIVANT and HAJNAL ANDRÉKA | 38 |
| Fixed point logics. by ANUJ DAWAR and YURI GUREVICH | 65 |

REVIEWS

| | |
|--|-----|
| S. Shapiro. <i>Thinking about mathematics</i> . Reviewed by Mark Balaguer | 89 |
| H. Woodin. <i>The axiom of determinacy, forcing axioms, and the nonstationary ideal</i> . Reviewed by Paul B. Larson | 91 |
| Four papers on descriptive set theory. Reviewed by Howard S. Becker | 94 |
| Papers from <i>Advances in modal logic, Volume 1</i> . Reviewed by Edwin D. Mares | 95 |
| F. Sommers and G. Englebretsen. <i>An invitation to formal reasoning</i> . Reviewed by William C. Purdy | 97 |
| A. Urquhart. <i>The complexity of linear logic with weakening</i> . Reviewed by Herman Ruge Jervell | 100 |
| R. Epstein and W. Carnielli. <i>Computability</i> . Reviewed by Carlos Augusto Di Prisco | 101 |
| M. Davis. <i>Engines of logic</i> . Reviewed by John W. Dawson | 104 |
| A. Feferman. <i>From Trotsky to Gödel</i> . Reviewed by H. B. Enderton | 104 |

IN MEMORIAM

| | |
|---|-----|
| In memoriam: Willard van Orman Quine. 1908–2000. by DAGFINN FØLLESDAL and CHARLES PARSONS | 105 |
|---|-----|

(Continued on inside back cover)

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Continued on inside back cover

equivalent to the hypothesis of measurable cardinal existence. The question is considered: for what nonstandard bounded t all points of X are t -nearstandard [4], where X is a non-rarefied compact. It is shown that any standard set including such t must be measurable. Moreover, for such t all t -standard points of X are standard.

[1] VLADIMIR KANOVI, *Unsolvability hypothesis in the internal set theory of Edward Nelson*, *Russian Mathematical Surveys*, vol. 46 (1991), no. 6, pp. 3–50.

[2] EDWARD NELSON, *Internal set theory: a new approach to nonstandard analysis*, *Bulletin of the American Mathematical Society*, vol. 83 (1977), no. 6, pp. 1165–1198.

[3] MARINA PROKHOROVA, *External analog of the choice axiom in IST*, IMM UrBr Russian Academy of Sciences, Ekaterinburg, 1999, 23 pp., Dep. VINITI 29.10.99, N 3246-B99.

[4] ———, *On the relative near-standardness in IST*, *Siberian Mathematical Journal*, vol. 39 (1998), no. 3, pp. 518–521.

- ▶ SAEED SALEHI, *Unprovability of Herbrand consistency in weak arithmetics*.
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By Herbrand's theorem, a theory T is consistent if and only if every finite set of its Skolem instances is (propositionally) satisfiable.

Herbrand Consistency of a theory T can be formalized as: “for any set of terms, say Λ , there is an evaluation on Λ which satisfies all the available Skolem instances of T ”.

Provability or unprovability of Herbrand Consistency of weak arithmetics (i.e., proper fragments of $I\Delta_0 + Exp$) in themselves had been an open problem since 1981 (see [2]).

In this paper, we modify the above definition such that its negation gives a *real* Herbrand proof of contradiction, even when Exp is not available and show the unprovability of Herbrand Consistency of $I\Delta_0$ (with the modified definition which is implied by the old one) in itself.

This incompleteness theorem has been shown for $I\Delta_0 + \Omega_2$ by Adamowicz [1], also in another unpublished paper, for $I\Delta_0 + \Omega_1$.

[1] Z. ADAMOWICZ and P. ZBIERSKI, *On Herbrand type consistency in weak theories*, to appear in *Archive for Mathematical Logic*.

[2] J. PARIS and A. WILKIE, Δ_0 -sets and induction, in *Proceedings of the Jadwisin Logic Conference, Poland*, Leeds University Press, 1981, pp. 237–248.

- ▶ PETER M. SCHUSTER, *Strong versus uniform continuity: a constructive round*.
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In the setting of constructive mathematics à la Bishop, uniform continuity is related to strong continuity—that is, the second-order functionality notion naturally associated with apartness spaces. The guiding questions are whether strong continuity in general differs from uniform continuity, and—supposing that it be so—under which circumstances they coincide, and in which instances they are interchangeable.

A mapping between metric spaces is called strongly continuous provided that if the images of two subspaces lie apart from each other, then so do the original subspaces. Every such mapping is pointwise continuous; more specifically, the pointwise continuity of any mapping requires the defining condition of strong continuity only from pairs of subspaces that contain at least one singleton. Moreover, any mapping is uniformly continuous if and only if its two-fold cartesian product is strongly continuous; whence strong continuity follows from uniform continuity, with which it is classically equivalent.

The well-known constructive intermediate value theorems, usually proved for uniformly

Articles should be expository or survey papers of broad interest that are accessible to a wide audience of logicians. They may deal with any areas of logic including mathematical or philosophical logic, logic in computer science or linguistics, the history or philosophy of logic, and applications of logic to other fields.

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