Hello!

THEOREMIZING PARADOXES: Turning Puzzles into Proofs

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Talk I: Paradoxes and their Theorems
1 June 2016

Outline

- Talk I:
  Paradoxes and their Theorems 1 June 2016
- Talk II:
  Theoremizing Yablo's Paradoxes 1 June 2016

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The Liar Paradox

$\mathfrak{L}$: The Sentence $\mathfrak{L}$ is Untrue.

Or, $\mathfrak{L}$ is True IF AND ONLY IF $\mathfrak{L}$ is Untrue. So, $\mathfrak{L} \Longleftrightarrow \neg\mathfrak{L}$

$\texttt{Propositional Logic} \vdash \neg(p \longleftrightarrow \neg p)$.

Theorem (Tarski)

If all the formulas can be coded by some terms in a language $\mathcal{L}$ ($\#: \mathcal{L}\texttt{-Formulas} \to \mathcal{L}\texttt{-Terms}, \varphi \mapsto \#\varphi$) and the diagonal lemma holds for a consistent $\mathcal{L}$-theory T (for any $\Psi(x) \in \mathcal{L}\texttt{-Formulas}$ there is some $\psi \in \mathcal{L}\texttt{-Sentences}$ such that $T \vdash \psi \leftrightarrow \Psi(\#\psi)$) then there can be no TRUTH PREDICATE *in $\mathcal{L}$ for T (an $\mathcal{L}$-formula $\mathfrak{T}(x)$ such that for any $\varphi \in \mathcal{L}\texttt{-Sentences}$, $T \vdash \varphi \leftrightarrow \mathfrak{T}(\#\varphi)$).*

Proof.

Take $\mathfrak{L}$ to be the diagonal sentence of $\neg\mathfrak{T}(x)$. Then

$T \vdash \mathfrak{L} \longleftrightarrow \neg\mathfrak{T}(\#\mathfrak{L}) \longleftrightarrow \neg\mathfrak{L}$ ⁎ ❑

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Russell's Paradox

The Set of All Sets that are not Members of Themselves.

Is This Set a Member of Itself or not?

Theorem (Invalidity of "unrestricted" Comprehension Principle)

For some formula $\varphi(x)$ there can be no set as $\{x \mid \varphi(x)\}$.

Proof.

Let $\varphi(x) =$"$x \notin x$". □

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$\varphi(x) =$"$\exists y\big[x = \mathscr{P}(y) \wedge x \notin y\big]$"

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Russell's Paradox—Theoremized

$$\texttt{Set Theory} \vdash \neg\exists y \forall x \big(x \in y \longleftrightarrow x \notin x\big).$$

Indeed, the proof does not make any essential use of $\in$.
Any binary relation will do:

$$\texttt{First-Order Logic} \vdash \neg\exists y \forall x \big(\Re(x,y) \longleftrightarrow \neg\Re(x,x)\big).$$

Russell's Popularization of his paradox:

Barber's Paradox

Shaves All and Only Those Who Cannot Shave Themselves.

$$\texttt{Second-Order Logic} \vdash \neg\exists Z^{(2)} \exists y \forall x \big(Z_{(x,y)} \longleftrightarrow \neg Z_{(x,x)}\big).$$

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Russell's Paradox vs. the Liar's

Russell's Paradox $\equiv$

$$\neg\exists y\forall x\big(\Re(x,y)\longleftrightarrow\neg\Re(x,x)\big)\equiv$$

$$\forall y\exists x\,\neg\big(\Re(x,y)\longleftrightarrow\neg\Re(x,x)\big)\equiv$$

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Russell's Paradox and Self-Reference

B. RUSSELL, On Some Difficulties in the Theory of Transfinite Numbers and Order Types, *Proceedings of the London Mathematical Society* 4:1 (1907) 29–53.

Given a property ϕ and a function f, such that, if ϕ belongs to all the members of u $[\forall x \in u : \phi(x)]$, $f`u$ $[f(u)]$ always exists, has the property $\phi$, and is not a member of $u$ $[f(u)\downarrow \in \{x \mid \phi(x)\} \setminus u]$; then the supposition that there is a class $w$ of all terms having the property $\phi$ $[w = \{x \mid \phi(x)\}]$ and that $f`w$ exists $[f(w)\downarrow]$ leads to the conclusion that $f`w$ both has and has not the property ϕ $[\phi(f(w)) \& \neg\phi(f(w))]$.

This generalization is important, because it covers all the contradictions [paradoxes] that have hitherto emerged in the subject.

Russell and Self-Reference

$$u \subseteq \{x \mid \phi(x)\} \Longrightarrow f(u)\!\downarrow\in \{x \mid \phi(x)\} \setminus u$$
$$w = \{x \mid \phi(x)\} \,\&\, f(w)\!\downarrow \Longrightarrow \phi(f(w)) \,\&\, \neg\phi(f(w))$$

Definition (Productive)

A set A is *productive*, if there exists a (partial) computable function $f:\mathbb{N}\to\mathbb{N}$ such that for every n, if $\mathcal{W}_n$ (the n-th RE set) is a subset of A, then $f(n)\!\downarrow\in A\setminus\mathcal{W}_n$. $\mathcal{W}_n \subseteq A \Longrightarrow f(n)\!\downarrow\in A\setminus\mathcal{W}_n$

Creative: a SEMI-DECIDABLE set whose complement is *productive*.

E. L. POST, Recursively Enumerable Sets of Positive Integers and their Decision Problems, *Bulletin of the American Mathematical Society* 50:5 (1944) 284–316.

"... every symbolic logic is incomplete ... . The conclusion is unescapable that even for such a fixed, well defined body of mathematical propositions, *mathematical thinking is, and must remain, essentially creative.*"

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Origin(s) of RUSSELL's (and others') Paradox(es)

J.A. COFFA, The Humble Origins of Russell's Paradox, *Russell* 33–34 (1979) 31–37.

On several occasions Russell pointed out that the discovery of his celebrated paradox concerning the class of all classes not belonging to themselves was intimately related to Cantor's proof that there is no greatest cardinal.

J. FRANKS, Cantor's Other Proofs that $\mathbb{R}$ is Uncountable, *Math. Magazine* 83:4 (2010) 283–289.

CANTOR's 3rd Proof for the Uncountability of $\mathbb{R}$
Could Also Show that $A \not\approx \mathscr{P}(A)$ (or $\mathscr{P}(A) \not\preccurlyeq A$).

CANTOR'S DIAGONAL ARGUMENT

For an $F : A \to \mathscr{P}(A)$ put $D_F = \{a \in A \mid a \notin F(a)\}$. Then

$$x \in D_F \longleftrightarrow x \notin F(x)$$

and so $D_F \neq F(\alpha)$ for any $\alpha \in A$:
if $D_F = F(\alpha)$ then $\alpha \in D_F \longleftrightarrow \alpha \notin F(\alpha) \longleftrightarrow \alpha \notin D_F$! ❑

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Origin(s) of RUSSELL's (and others') Paradox(es)

J.A. COFFA, The Humble Origins of Russell's Paradox, *Russell* 33–34 (1979) 31–37.

On several occasions Russell pointed out that the discovery of his celebrated paradox concerning the class of all classes not belonging to themselves was intimately related to Cantor's proof that there is no greatest cardinal.

J. FRANKS, Cantor's Other Proofs that $\mathbb{R}$ is Uncountable, *Math. Magazine* 83:4 (2010) 283–289.

CANTOR's 3rd Proof for the Uncountability of $\mathbb{R}$
Could Also Show that $A \not\cong \mathscr{P}(A)$ (or $\mathscr{P}(A) \not\preccurlyeq A$).

CANTOR'S DIAGONAL ARGUMENT

For an $F: A \to \mathscr{P}(A)$ put $D_F = \{a \in A \mid a \notin F(a)\}$. Then

$$x \in D_F \longleftrightarrow x \notin F(x)$$

and so $D_F \neq F(\alpha)$ for any $\alpha \in A$:
if $D_F = F(\alpha)$ then $\alpha \in D_F \longleftrightarrow \alpha \notin F(\alpha) \longleftrightarrow \alpha \notin D_F$! ❑

Diagonal Argument and Self-Reference

K. SIMMONS, The Diagonal Argument and the Liar, *J. Philosophical Logic* 19:3 (1990) 277–303.

There are arguments found in various areas of mathematical logic that taken to form a family: the family of *diagonal arguments*. Much of recursion theory may be described as a theory of diagonalization; diagonal arguments establish basic results of set theory; and they play a central role in the proofs of limitative theorems of Gödel and Tarski. Diagonal arguments also give rise to set-theoretical and semantical paradoxes.

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I dedicate this essay to the two-dozen-odd people whose refutations of Cantor's diagonal argument … have come to me either as a referee or an editor in the last twenty years or so. … A few years ago it occurred to me to wonder why so many people devote so much energy to refuting this harmless little argument—what had it done to make them so angry with it?

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Ubiquity of the Diagonal Argument

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In Economics:

R.P. MURPHY, Cantor's Diagonal Argument: An Extension to the Socialist Calculation Debate, *The Quarterly J. of Australian Economics* 9:2 (2006) 3–11.

In Physics:

▷ D.H. WOLPERT, Physical Limits of Inference, *Physica D* 237 (2008) 1257–1281.

▷ P.-M. BINDER, Theories of Almost Everything, *Nature* 455 (2008) 884–885.

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Ongoingness of The Diagonal Argument

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Diagonal Argument Again (1)

QUINE's Proof (inconsistency of comprehension principle)

$$Q_n = \{x \mid \neg\exists z_1, \cdots, z_n[x \in z_n \in z_{n-1} \in \cdots \in z_1 \in x]\}$$

W. V. QUINE, *Mathematical Logic*, Harvard University Press (2nd ed. 1981).

Proof.

$Q_n \in Q_n \longrightarrow$

$$\exists z_1, \cdots, z_n(= Q_n)[Q_n \in z_n \in z_{n-1} \in \cdots \in z_1 \in Q_n]$$

$$\longrightarrow Q_n \notin Q_n.$$

contradiction !

$$Q_n \notin Q_n \longrightarrow \exists z_1, \cdots, z_n[Q_n \in z_n \in z_{n-1} \in \cdots \in z_1 \in Q_n]$$

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For $n = 0$: RUSSELL's Proof (and Paradox).

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W. V. Quine, *Mathematical Logic*, Harvard University Press (2nd ed. 1981).

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contradiction ! ☐

For $n = 0$: Russell's Proof (and Paradox).

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Diagonal Argument Again (2)

$$Q_n = \{x \mid \neg\exists z_1, \cdots, z_n[x \in z_n \in z_{n-1} \in \cdots \in z_1 \in x]\}$$

Z. Šikić, Cantor's Theorem and Paradoxical Classes, *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 32:13-16 (1986) 221–226.

Proof.

$$\{\!\{A\}\!\}^n = \underbrace{\{\{\cdots\{}_{n\text{–times}} A \overbrace{\}\}\cdots\}}^{n\text{–times}}$$

$$\begin{aligned}\{\!\{Q_n\}\!\}^n \notin Q_n \longleftrightarrow \\ &\exists z_1, \cdots, z_n[\{\!\{Q_n\}\!\}^n \in z_n \in z_{n-1} \in \cdots \in z_1 \in \{\!\{Q_n\}\!\}^n] \\ \longleftrightarrow\ &\exists z_1, \cdots, z_n[\{\!\{Q_n\}\!\}^n \in z_n \wedge \textstyle\bigwedge_{j=1}^{n} z_j = \{\!\{Q_n\}\!\}^{n-j}] \\ \longleftrightarrow\ &\exists z_n[\{\!\{Q_n\}\!\}^n \in z_n \wedge z_n = Q_n] \longleftrightarrow \{\!\{Q_n\}\!\}^n \in Q_n.\end{aligned}$$

❑

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$$\longleftrightarrow \exists z_1, \cdots, z_n[\{\!\!\{Q_n\}\!\!\}^n \in z_n \wedge \bigwedge_{j=1}^{n} z_j = \{\!\!\{Q_n\}\!\!\}^{n-j}]$$

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Diagonal Argument Again (3)

The Paradox of Well–Founded Sets

Sets Whose Every Membership Chain Finitely Terminates

$$Q_\infty = \{x \mid \neg\exists z_1, z_2, \cdots [\cdots \in z_2 \in z_1 \in x]\}$$

- $Q_\infty \in Q_\infty \longrightarrow \cdots \in Q_\infty \in Q_\infty \in Q_\infty \in Q_\infty$
  $\longrightarrow \exists z_1, z_2, \cdots [\cdots \in z_2 \in z_1 \in Q_\infty]$
  $\longrightarrow Q_\infty \notin Q_\infty$

- $Q_\infty \notin Q_\infty \longrightarrow \exists z_1, z_2, \cdots [\cdots \in z_2 \in z_1 \in Q_\infty]$
  $\longrightarrow \exists z_1 \Big(\exists z_2, z_3 \cdots [\cdots \in z_3 \in z_2 \in z_1] \wedge z_1 \in Q_\infty\Big)$
  $\longrightarrow \exists z_1 \big(z_1 \notin Q_\infty \wedge z_1 \in Q_\infty\big) \longrightarrow$ contradiction !

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Other Proofs of Cantor's Theorem.

For $F : A \to \mathscr{P}(A)$ Let

$$D_F^\infty = \{x \in A \mid \neg\exists z_1, z_2, \cdots [\bigwedge_{i=1}^{\infty}(z_{i+1} \in F(z_i)) \wedge z_1 \in F(x)]\}$$

and

$$D_F^n = \{x \mid \neg\exists z_1, \cdots, z_n [x \in F(z_n) \wedge \bigwedge_{i=1}^{n-1}(z_{i+1} \in F(z_i)) \wedge z_1 \in F(x)]\}.$$

One can show that none of these can be in the range of F:

$$D_F^\infty \subseteq \cdots \subseteq D_F^{n+1} \subseteq D_F^n \subseteq \cdots \subseteq D_F^0 = D_F \quad \square$$

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Other Proofs of CANTOR's Theorem.

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Paradoxes and Self-Reference / Circularity

A General Belief:

all the paradoxes involve self-reference / circularity
(in a way or another).

YABLO's Paradox $Y_1, Y_2, Y_3, \cdots$

For all n, Y_n is True if and only if All Y_k's for $k > n$ are Untrue.

$Y_1 :$ $Y_2, Y_3, Y_4, \cdots$ are all untrue.
$Y_2 :$ $Y_3, Y_4, Y_5, \cdots$ are all untrue.
$Y_3 :$ $Y_4, Y_5, Y_6, \cdots$ are all untrue.
$\vdots$

- If some Y_m is true, then $Y_{m+1}, Y_{m+2}, Y_{m+3}, \cdots$ are all untrue. Whence Y_{m+1} is untrue but also true (by $\bigwedge_{i \geqslant m+2} Y_i$).
- If all Y_k's are untrue, then $Y_0, Y_1, Y_2, \cdots$ are true!

Paradoxes and Self-Reference / Circularity

A General Belief:

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(in a way or another).

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See You Later

To Be Continued ...

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Thank You!

Thanks to

The Participants .... For Listening · · ·

and

The Organizers — For Taking Care of Everything · · ·

SaeedSalehi.ir

Hello!

Theoremizing Paradoxes: Turning Puzzles into Proofs*

Saeed Salehi

University of Tabriz & IPM

http://SaeedSalehi.ir/

*A Joint Work with Ahmad Karimi.

SWAMPLANDIA 2016, Ghent University
Talk II: Theoremizing Yablo's Paradoxes
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(always)	$\mathcal{Y}_n \iff \forall i > n\, (\mathcal{Y}_i \ \texttt{is untrue})$
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J. KETLAND, Yablo's Paradox and ω-Inconsistency, *Synthese* 145:3 (2005) 295–302.

$$\{\forall x\exists y(x<y), \forall x,y,z(x<y<z \rightarrow x<z)\} \vdash \neg\forall x\big(\varphi(x) \leftrightarrow \forall y[x<y \rightarrow \neg\varphi(y)]\big).$$

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Theorem (First-Order Logic)

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Theoremizing YABLO's Paradox (1)

J. KETLAND, Yablo's Paradox and ω-Inconsistency, *Synthese* 145:3 (2005) 295–302.

$$\{\forall x \exists y (x<y), \forall x,y,z(x<y<z \to x<z)\} \vdash \neg\forall x\big(\varphi(x) \leftrightarrow \forall y[x<y \to \neg\varphi(y)]\big).$$

More generally,

Theorem (First-Order Logic)

$$\forall x \exists y \big(x \Re y \wedge \forall z[y \Re z \to x \Re z]\big) \vdash \neg\forall x\big(\varphi(x) \leftrightarrow \forall y[x \Re y \to \neg\varphi(y)]\big)$$

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Theoremizing Yablo 's Paradox (2)

Theorem (Second-Order Logic)

$\forall x \exists y \big(x\Re y \wedge \forall z[y\Re z \rightarrow x\Re z]\big) \vdash \neg\exists Z^{(1)} \forall x \big(Z_x \leftrightarrow \forall y[x\Re y \rightarrow \neg Z_y]\big)$

Definition (Yablo System)

Let us call a directed graph $\langle A; R\rangle$ (with $R \subseteq A^2$) a Yablo system when $\neg\exists Z^{(1)} \forall x \big(Z_x \leftrightarrow \forall y[xRy \rightarrow \neg Z_y]\big)$.

example Any odd-cycle, such as $\langle\{a\}; \{a\Re a\}\rangle$. The Liar's Paradox

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Yablo's Paradox — 1st or 2nd Order? (1)

The first-order condition $\forall x \exists y \big(x \Re y \wedge \forall z [y \Re z \rightarrow x \Re z]\big)$ (and many more weaker conditions) imply the Yablo-ness of the graph.

Theorem (Nonfirstorderizability of Yabloness)

The Yabloness $\neg\exists Z^{(1)} \forall x \big(Z_x \leftrightarrow \neg\exists y [x \Re y \wedge Z_y]\big)$ is not equivalent to any first-order formula (in the language $\langle \Re \rangle$).

https://en.wikipedia.org/wiki/Nonfirstorderizability

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there is no group which contains all and only those whose no related one is (already) in the group

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Conjecture (Any Help is Appreciated!)

The YABLOness $\neg\exists Z^{(1)}\forall x\big(Z_x \leftrightarrow \neg\exists y[x\Re y \wedge Z_y]\big)$ is not equivalent to any first-order $\langle\Re\rangle$-theory, either.

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Yablo's Paradox — 1st or 2nd Order? or non?

Is That It?

L. M. Picollo, Yablo's Paradox in Second-Order Languages: Consistency and Unsatisfiability, *Studia Logica* 101:3 (2013) 601–617.

If we embrace the second-order notion of logical consequence we must subscribe to the idea that the second-order calculus is not powerful enough for representing Yablo's argument, and neither is the first-order calculus.

Is there a better (or just another) logic that represents Yablo's Paradox (and his argument)?

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Linear Temporal Logic (syntax)

(Propositional) Linear Temporal Logic (LTL):

$\bigcirc$ **Next** $\qquad \square$ **Always (from now on)**

Formulas: p (atomic) | $\neg\varphi$ | $\varphi_1 \wedge \varphi_2$ | $\varphi_1 \vee \varphi_2$ | $\varphi_1 \rightarrow \varphi_2$ | $\bigcirc\varphi$ | $\square\varphi$

$\neg\bigcirc\varphi$: not in the next step φ

$\bigcirc\neg\varphi$: in the next step not φ

$\bigcirc\square\varphi$: in the next time always (from then on) φ

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LTL and Yablo's Paradox

Yablo's Paradox:

"everyone in an infinite linear row claims that all the forthcoming ones are lying"

$\varphi \longleftrightarrow \bigcirc\Box\neg\varphi \qquad (\equiv \Box\bigcirc\neg\varphi) \quad (\equiv \Box\neg\bigcirc\varphi)$

"I will always deny all my future (from the next step onward) sayings"

"I will always deny whatever I will have said afterwards"

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Linear Temporal Logic (semantics)

(Propositional) Linear Temporal Logic (LTL):

$\bigcirc$: **Next** $\Box$: **Always** $\Diamond$: **Sometime**

The Intended Model: $\langle \mathbb{N}, \Vdash \rangle$ where $\Vdash \subseteq \mathbb{N} \times \texttt{Atoms}$ can be extended to all formulas by:

- $n \Vdash \varphi \wedge \psi$ iff $n \Vdash \varphi$ and $n \Vdash \psi$
- $n \Vdash \neg\varphi$ iff $n \nVdash \varphi$
- $n \Vdash \bigcirc\varphi$ iff $(n+1) \Vdash \varphi$
- $n \Vdash \Box\varphi$ iff $m \Vdash \varphi$ for every $m \geq n$

An Example of a Law of LTL: $\Box\bigcirc\varphi \equiv \bigcirc\Box\varphi$

$n \Vdash \Box\bigcirc\varphi$ iff $\forall x \geq n\big[x \Vdash \bigcirc\varphi\big]$ iff $\forall x \geq n\big[(x+1) \Vdash \varphi\big]$
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$$\Diamond\varphi = \neg\Box\neg\varphi$$

Another Law of LTL: $\bigcirc\neg\varphi \equiv \neg\bigcirc\varphi$

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YABLO's Paradox as an LTL–Theorem

A. KARIMI & S. SALEHI, Diagonal Arguments and Fixed Points, *Bulletin of the Iranian Mathematical Society*, to appear.

Theorem (YABLO's Paradox $\Longrightarrow$ Genuine Theorem)

(Propositional) Linear Temporal Logic $\models \neg\Box(\varphi \longleftrightarrow \bigcirc\Box\neg\varphi)$.

Proof.

If $n \Vdash \Box(\varphi \leftrightarrow \bigcirc\Box\neg\varphi)$ for some model, then

$$\forall i \geq n : i \Vdash \varphi \iff i \Vdash \bigcirc\Box\neg\varphi \iff i+1 \Vdash \Box\neg\varphi.$$

(i) If for some $j \geq n$ we have $j \Vdash \varphi$, then $j+1 \Vdash \Box\neg\varphi$ and so $j+\ell \nVdash \varphi$ for all $\ell \geq 1$. In particular, $j+1 \nVdash \varphi$ whence $j+2 \nVdash \Box\neg\varphi$ which is in contradiction with $j+1 \Vdash \Box\neg\varphi$.

(ii) If for all $j \geq n$ we have $j \nVdash \varphi$, then $n \nVdash \varphi$ so $n+1 \nVdash \Box\neg\varphi$; hence there must exist some $i > n$ with $i \Vdash \varphi$ which contradicts (i) ! ❑

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Yablo's Paradoxes as LTL–Theorems

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$(\longleftrightarrow \Box\bigcirc\Diamond\neg\mathscr{Y} \longleftrightarrow \Box\Diamond\bigcirc\neg\mathscr{Y} \longleftrightarrow \Box\Diamond\neg\bigcirc\mathscr{Y}).$

YABLO's Paradoxes as LTL–Theorems

A. KARIMI & S. SALEHI, Theoremizing Yablo's Paradox, *arXiv:1406.0134 [math.LO]*, http://arxiv.org/abs/1406.0134

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$\mathrm{LTL} \models \neg\Box(\varphi \longleftrightarrow \bigcirc\Box\neg\varphi)$.

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Axioms:
- All the Propositional Tautologies

(LTL1) $\neg\bigcirc\varphi \longleftrightarrow \bigcirc\neg\varphi$

(LTL2) $\bigcirc(\varphi \to \psi) \longrightarrow (\bigcirc\varphi \to \bigcirc\psi)$

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Rules: (MP)

$$\frac{\varphi, \quad \varphi \to \psi}{\psi}$$

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- $\Box\Diamond\Box A \longleftrightarrow \Diamond\Box A$
- $A \to \bigcirc A \;/\; A \to \Box A$
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More (Non-)Fixedpoints of LTL

A. KARIMI & S. SALEHI, Theoremizing Yablo's Paradox,
arXiv:1406.0134 [math.LO], http://arxiv.org/abs/1406.0134

Proposition

The operators $x \mapsto \neg\Box x$ *and* $x \mapsto \Box\neg x$ *do not have any fixed–points in* LTL*; i.e.,* $\mathrm{LTL} \models \neg\Box(\varphi \leftrightarrow \neg\Box\varphi)$ *and* $\mathrm{LTL} \models \neg\Box(\varphi \leftrightarrow \Box\neg\varphi)$.

Proof.

If $n \Vdash \Box(\varphi \leftrightarrow \Box\neg\varphi)$, then for any $i \geqslant n$ we have $i \Vdash \varphi \Leftrightarrow i \Vdash \Box\neg\varphi$. Now, by $\models \Box\neg\varphi \rightarrow \neg\varphi$ we have $i \Vdash \varphi \Rightarrow i \Vdash \neg\varphi$, so $i \Vdash \neg\varphi$ for all $i \geqslant n$. Thus, in particular $n \Vdash \neg\varphi$, and also $n \Vdash \Box\neg\varphi$, ⋇! ❑

Remark

Some other operators like $x \mapsto \Box x$ *or* $x \mapsto \neg\bigcirc x$ *do have fixed–points;* $\langle \mathfrak{t}, \mathfrak{t}, \mathfrak{t}, \mathfrak{t}, \mathfrak{t}, \mathfrak{t}, \cdots \rangle$ *for the former and* $\langle \mathfrak{f}, \mathfrak{t}, \mathfrak{f}, \mathfrak{t}, \mathfrak{f}, \mathfrak{t}, \cdots \rangle$ *for the latter.* ✧

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See You Later

THAT WAS FOR NOW ...

- Talk I:
  Paradoxes and their Theorems — 1 June 2016
- Talk II:
  Theoremizing Yablo's Paradoxes — 1 June 2016

Thank You!

Thanks to

The Participants .... For Listening ···

and

The Organizers — For Taking Care of Everything ···

SaeedSalehi.ir