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Automatic presentations for semigroups. (English summary)
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The paper studies finitely generated semigroups that admit finite automata presentations. It is shown that all finitely generated commutative semigroups are FA-presentable. Then finitely generated cancellative semigroups that are FA-presentable are characterized: a finitely generated cancellative semigroup is FA-presentable if and only if it can be embedded into a virtually abelian group. A group is called virtually abelian if it has an abelian subgroup of finite index. The notions of FA-presentable and automatic semigroups are carefully distinguished from one another, and some relations between these classes for finitely generated semigroups and groups are obtained. It is also noted that any cancellative semigroup that admits a unary (i.e., over a one-word alphabet) automatic presentation is finite. *Saeed Salehi*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.